Determining the Sign of Δ_{31} by Future Long Baseline and Reactor Experiments

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Abstract

We study the determination of neutrino mass hierarchy, through neutrino experiments within the next ten years. The T2K neutrino oscillation experiment will start in 2009. In the experiment the high intensity ν_μ beam from JHF is directed to Super-Kamiokande (SK) detector 295 km away. The $NO\nu A$ (off axies neutrino oscillation) experiment is being planned, with the ν_μ beam from Fermi-Lab directed to a site 610km away, which is 0,7,14 milliradian off-axies. Both the above experiments will measure $\nu_\mu \to \nu_e$ oscillation probability. The double-CHOOZ experiment under construction detects $\overline{\nu_e} s$ emitted by nuclear reactors both through a near detector (150m) and a far detector (1.05 Km) and measure $\overline{\nu_e} \to \overline{\nu_e}$ survival probability. In this paper, we outline a procedure to determine the sign of Δ_{31} from the simulated data of the above experiments.

1 Introduction

Recent advance in neutrino physics observation mainly of astrophysical observation suggested the existence of tiny neutrino mass. The experiment and observation have shown evidences for neutrino oscillation. The Solar neutrino deficit has long been observed [1,2,3,4), the atmospheric neutrino anomaly has been indicates that neutrino are massive and there is mixing in lepton sector and currently almost confirmed by KamLAND [8] and hence indicates that neutrino are massive and there is mixing in lepton sector. Since there is mixing in lepton sector, this indicate to imagine that there occurs CP violation effect in lepton sector. Several physicist have considered whether we can see CP violation effects in lepton sector through long baseline oscillation experiments. The neutrino oscillation probabilities, in general depends on six parameter two independent mass squared difference Δ_{21} and Δ_{31} , there mixing θ_{12}, θ_{23} and θ_{13} and one CP violating phase δ. There are two large mixing angle (θ_{12}, θ_{23})

and one small (θ_{13})angle, and two mass square difference, $\Delta_{ij} = m_j^2 - m_j^2$, with m_{ij} is the neutrino masses,

where

$$\Delta_{21} = \Delta_{solar,} \tag{1}$$

$$\Delta_{31} = \Delta_{Atmo.} \tag{2}$$

The sign of $\Delta_{31}$and of θ_{23} when $\theta_{23} \neq$0, can not be determine with the existing data. For the mass square difference, there are two possibility, $\Delta_{31} > 0$ or $\Delta_{31} < 0$, correspond to two different types of neutrino mass order normal mass hierarchy, $m_1 < m_2 < m_3$ ($\Delta_{31} > 0$), and inverted hierarchy, $m_1 > m_2 > m_3(\Delta_{31} < 0)$. The angles $\theta_{12}$and θ_{23} represent the neutrino mixing angles corresponding to solar and atmospheric neutrino oscillation, much progress has been made to words, determining the values of the three mixing angle. From measurement of the neutrino survival probability $\nu_\mu \to \nu_e$ and $\nu_e \to \nu_e$ in the atmospheric flux, so that one mixing angle is near $\frac{\pi}{4}$and one is small [11] from the $\nu_e \to \nu_e$ survival probability in the solar flux, so that the mixing angle is either large (LMA) or small (SMA) by solar solution [12]. Nothing is known about CP violating phase. In this paper, we tested the sign of $\Delta_{31}$by using three different baseline (T2K, $NO\nu A$ and Double CHOOZ) experiments. The purpose of this paper is to determine the sign of Δ_{31} by χ^2 analysis. Section 2 describes the mixing angle and masses difference. Section 3 describes the mass hierarchy effect in $\nu_\mu \to \nu_e$oscillation probability. In Sec-4. Determine the sign of Δ_{31} by χ^2analysis. Section 5 summarizes the results and conclusions.

2 Mixing Angles and Neutrino Mass Squared Differences

The first evidence is the observation of zenith-angle dependence of atmospheric neutrino defect [13] dependent of the atmospheric neutrino $\nu_\mu \to \nu_\mu$ transition with the mass difference and the mixing as

$$\Delta_{31} = (1-2) \times 10^{-3} eV^2, sin^2 2\theta_{23} = 1.0. \tag{3}$$

The second evidence is the solar neutrino deficit [14].Which is consistent with $\nu_\mu \to \nu_\tau/\nu_e$ transition. The SNO experiments [15] are consistent with the standard solar model [16] and strong suggest the LMA solution.

$$\Delta_{21} = 7 \times 10^{-5} eV^2, sin^2 2\theta_{12} = 0.8. \tag{4}$$

Solar neutrino experiments (Super-K, GALLEX, SAGE, SNO and GNO) show that neutrino oscillations, neutrino oscillation provide the most elegant explanation of all the data [17]..

$$\Delta_{solar} = 7^{+5}_{-1.3} \times 10^{-5} eV^2, \tag{5}$$

$$tan^2\theta_{solar} = 0.4^{+0.14}_{-0.1}. \tag{6}$$

Atmospheric neutrino experiments (Kamiokande, Super-K) also show that neutrino oscillation. The most excellent fit to the all data [17].

$$\Delta_{atmo} = 2.0^{+1.0}_{-0.92} \times 10^{-3} eV^2, \tag{7}$$

$$sin^2 2\theta_{atmo} = 0.4^{+0.14}_{-0.10}. \tag{8}$$

The CHOOZ reactor experiment [18] gives the upper bound of the third mixing angle θ_{13}as

$$sin^2\theta_{13} < 0.20 \qquad for \qquad |\Delta_{31}| = 2.0 \times 10^{-3} eV^2, \tag{9}$$

$$sin^2\theta_{13} < 0.16 \qquad for \qquad |\Delta_{31}| = 2.5 \times 10^{-3} eV^2,, \tag{10}$$

$$sin^2\theta_{13} < 0.14 \qquad for \qquad |\Delta_{31}| = 3.0 \times 10^{-3} eV^2, \tag{11}$$

at the 90 % CL. The CP phase δ has not been constrained. In future neutrino experiments, which plan to measure the oscillation parameter precisely.

3 Mass Hierarchy Effect in Neutrino Oscillation Probability

Let us briefly recall our present knowledge of neutrino oscillation parameters. There are three flavors of neutrinos and they mix to form three mass eigensates. This mixing is given by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{12}$$

where mixing matrix U parametrized [19] as

$$U = R(\theta_{23})\Pi R(\theta_{13})\Pi^* R(\theta_{12}). \tag{13}$$

In the above mixing matrix, Π is a diagonal matrix containing the CP violating phase δ and $R(\theta_{ij})$is is the form of rotation matrices. The mass eigenstates ν_i have eigenvalues m_i . Neutrino oscillation probabilities depend on the two mass squared differences $\Delta_{21} = m_2^2 - m_1^2$,$\Delta_{31} = m_3^2 - m_1^2$, the three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and the CP violating phase δ. Solar neutrino data and KamLAND experiment determine Δ_{21} and θ_{12}. Atmospheric neutrino data and K2K and MINOS experiments determine $|\Delta_{31}|$ and θ_{23}. CHOOZ experiment and solar neutrino data constrain $\theta_{13}$to be small. There is no information at present on the CP phase δ. The future experiments are expected to measured θ_{13} and

determine the sign of $\Delta_{31}$and the magnitude of CP violation, in addition to improving the precision of known neutrino oscillation parameters.

Double CHOOZ experiment is a reactor based experiment dedicated to measuring θ_{13}. In this experiment systematic errors are minimised by having identical near and far detectors at distances 150 meter and 1050 meter from the sources respectively. This experiment can measure non zero value of θ_{13} if $sin^2 2\theta_{13} \geq 0.05$[20]. Daya Bay reactor experiment will have a similar sensitivity [21]. In T2K experiment the high intensity of ν_μ beam from JPARC accelerator is directed to SK detector 295 Km away. The detector is 2^o off-axis from the beam, which lead to the neutrino flux peaking at lower energy. T2K is a very high statistics experiment that is expected to start taking data in 2009. The neutrino flux is about 100 times the flux of K2K. The number of ν_μ charged current events expected, in the case of no oscillation, is about 3100 per year. This experiment will improve the precession of Δ_{31} and θ_{23} by measuring the muon neutrino survival probability $P(\nu_\mu \rightarrow \nu_e)$. It can also measure θ_{13} through the measurement of $P_{\mu e}$._$NO\nu A$ is also an accelerator based experiment, which uses ν_μ beam from Fermilab 810 Km away. The detecting material in this experiment is a scintillator which gives it an excellent electron detection capability. Thus $NO\nu A$can make a precise determination of $P_{\mu e}$,$NO\nu A$ which is expected to start taking data in 2011, also will be placed at an off-axis location. Because of the longer distance the flux $NO\nu A$ is peaked at higher energy compared to that of T2K. Matter term, which is proportional to neutrino energy, causes a 25% change in $P_{\mu e}$ whereas the change in $P_{\mu e}$of T2K is only about 10% [22]. If Δ_{31} positive $P_{\mu e}$increases, whereas if Δ_{31} is negative it decreases. Below we describe a procedure by which sign of Δ_{31} can be determined using the data from Double CHOOZ, T2K and $NO\nu A$. We will compute the smallest value of θ_{13} for which the sign of Δ_{31} can be determined independent of the CP phase δ.

4 Mass Hierarchy Effect in $P^m_{\mu e}$_Oscillation Probability

- $P^m_{\mu e}$ with $\Delta_{21} = 0$

Neutrino oscillation probability, $\nu_\mu \rightarrow \nu_e$ in long base line experiments is modified by the propagation of neutrino through the matter of earth's crust [23]. It increases the oscillation probability for neutrinos if $\Delta_{31}$is positive and decreases it $\Delta_{31}$is negative. The reverse is true for anti-neutrinos. Here we consider a method of determining the sign of Δ_{31}using $\nu_\mu$beams only.

In three flavor mixing, $\nu_\mu \rightarrow \nu_e$ oscillation probability is given by

$$P_{\mu e} = sin^2\theta_{23} sin^2 2\theta_{13} sin^2 \left(\frac{1.27\Delta_{31} L}{E} \right), \tag{14}$$

where Δ_{31} in eV^2, the baseline L is in Km and the neutrino energy E is in GeV. In the above equation, we made the approximation of setting $\Delta_{21} =$

0, which made it independent of θ_{13} and the CP phase δ. In long baseline experiments, the neutrinos propagate through earth's crust which has constant density of about 3gm/cc. The oscillation probability modified by matter effect is given by

$$P^m_{\mu e} = sin^2\theta_{23} sin^2 2\theta^m_{13} sin^2\left(\frac{1.27\Delta^m_{31}L}{E}\right), \tag{15}$$

where

$$sin2\theta^m_{13} = \frac{\Delta_{31} sin2\theta_{13}}{\Delta^m_{31}}, \tag{16}$$

$$\Delta^m_{31} = \sqrt{(\Delta_{31}cos2\theta_{13} - A)^2 + (\Delta_{31}sin2\theta_{13})^2} \tag{17}$$

Here A is the matter term and is given by

$$A = 2\sqrt{2}G_F NeE = 0.76 \times 10^{-4}\rho(in\, gm/cc)E_\nu(in\, GeV). \tag{18}$$

From the expression of $P_{\mu e}$ in three flavor oscillations, we can compute the magnitude of the terms dependent on Δ_{21}. It turns out that as the CP violating phase δ varies from $-\pi$ to π, $P_{\mu e}$ changes by about 25 %. Therefore setting $\Delta_{21} = 0$ is not a good approximation for analyzing matter effects in long baseline experiments.

- $P^m_{\mu e}$ with $\Delta_{21} \neq 0$

Exact expression for $P_{\mu e}$ with matter effects is very complicated. The expression derived using a perturbation expansion with θ_{13} and $\alpha = \frac{\Delta_{21}}{\Delta_{31}}$ as a small parameters works very well for baselines up to 1000 Km [24, 25]. Carrying out the perturbation expansion to second order in the small parameters, the following analytic formula for $\nu_\mu \to \nu_e$ is obtained with the assumption of constant matter density

$$P^m_{\mu e} = sin^2 2\theta_{23}\frac{sin^2 2\theta_{13}}{(A_1-1)^2} sin^2((A_1-1)\Delta)$$

$$\pm\frac{\alpha sin\delta cos\theta_{13} sin2\theta_{13} sin2\theta_{12} sin2\theta_{23}}{A_1(1-A_1)} sin(\Delta) sin(A_1\Delta) sin((1-A_1)\Delta)$$

$$+\frac{\alpha cos\delta cos\theta_{13} sin2\theta_{13} sin2\theta_{12} sin2\theta_{23}}{A_1(1-A_1)} sin(\Delta) cos(A_1\Delta) sin((1-A_1)\Delta)$$

$$\frac{\alpha^2 cos^2\theta_{23} sin^2 2\theta_{12} sin^2(A_1\Delta)}{A_1^2}, \tag{19}$$

where $\alpha = \Delta_{21}/\Delta_{31}$,$\Delta = \Delta_{31}L/4E$,$A_1 = 2\sqrt{2}G_F NeE/\Delta_{31}$, G_F is the Fermic coupling constant and n_e is the electron density in earth's crust. We see

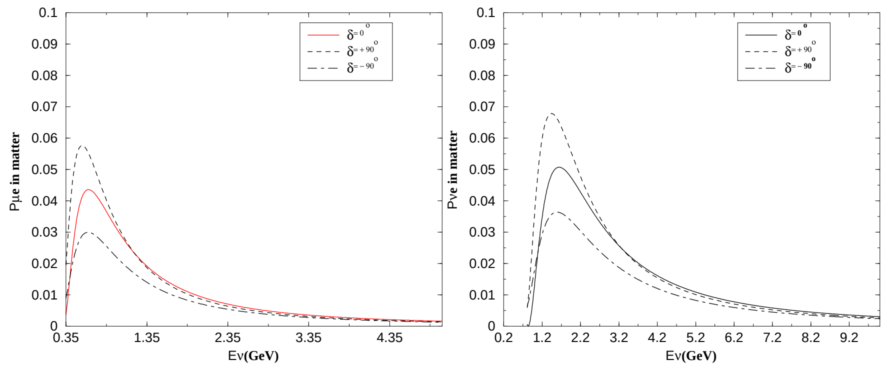

Figure 1: $P_{\mu e}$ oscillations probabilities vs E for $\Delta_{21} = 2.5 \times 10^{-3} eV^2$,$\theta_{13} = 8^o$,L=295 km and L=810 km. The middle line is $P^m_{\mu e}(\delta = 0^o)$,the upper line is $P^m_{\mu e}(\delta = +90^o)$ and lower line is $P^m_{\mu e}(\delta = -90^o)$.

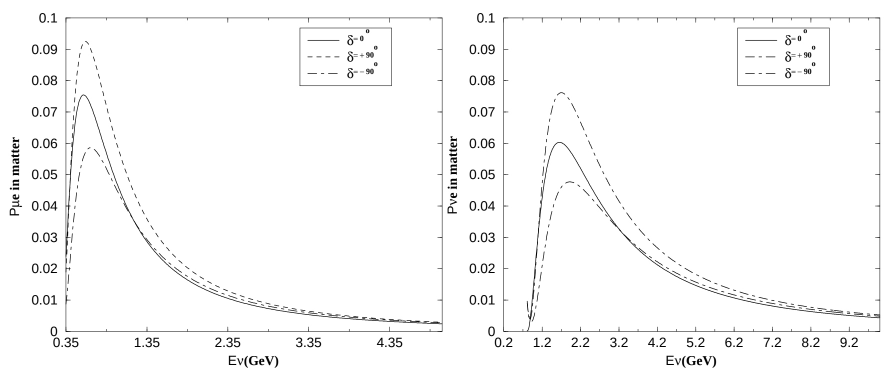

Figure 2: $P_{\mu e}$ oscillations probabilities vs E for $-2.5\times10^{-3} eV^2$,$\theta_{13} = 12^o$,L=295 km and L=810 km. The middle line is $P^m_{\mu e}(\delta = 0^o)$,the upper line is $P^m_{\mu e}(\delta = +90^o)$ and lower line is $P^m_{\mu e}(\delta = -90^o)$.

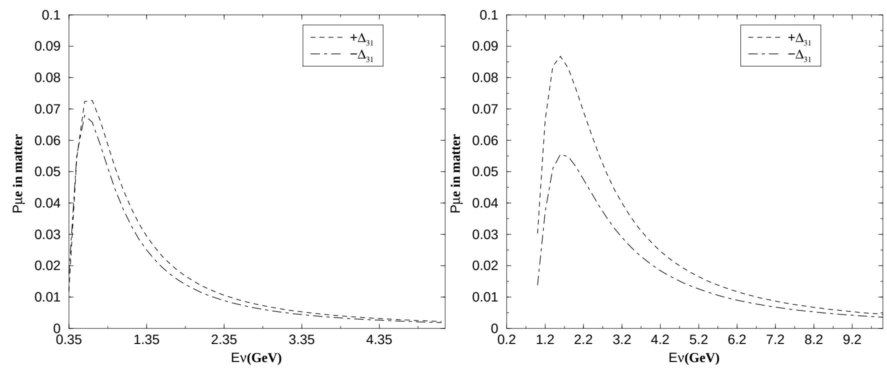

Figure 3: $P_{\mu e}$ oscillations probabilities vs E for $\Delta_{21} = 2.5 \times 10^{-3} eV^2$,$\theta_{13} = 10^o$,L=295 km and L=810 km. The dashed line is $P^m_{\mu e}(\delta = 30^o)$ with Δ_{31} positive and dot-dashed line is $P^m_{\mu e}(\delta = 75^o)$ with Δ_{31} negative.

the above expression depends on three unknown quantities θ_{13}, sign of $\Delta_{31}$and the CP phase δ.

There are two kinds of degeneracies inherent in the three flavor expression for $P^m_{\mu e}$. The first one occurs due to following reason. Since θ_{13} is unknown, the $P^m_{\mu e}$ for positive Δ_{31} and smaller θ_{13} can be essentially the same as the $P^m_{\mu e}$ for negative Δ_{31} and larger θ_{13} This is illustrated in fig(1) and fig(2). Precise determination of θ_{13} by Double CHOOZ can eliminate this degeneracy. There is a further degeneracy involving the CP phase δ . At present there is no experimental information on this phase. Note that Double CHOOZ is completely insensitive to δ. For a given long baseline experiment, it is possible to find two values of the CP phase, δ^+ and δ^-, such that $P^m_{\mu e}(+\Delta_{31}, \delta^+) = P^m_{\mu e}(-\Delta_{31}, \delta^-)$, with all other oscillation parameters, including θ_{13} fixed. This is illustrated in fig(3) and fig(4). However, the above degeneracy can occur for only one baseline length at a time. In fig(3) $P^m_{\mu e}$ for T2K is essentially the same for both signs of Δ_{31}but $P^m_{\mu e}$ can distinguish between the two signs of Δ_{31}. In fig(4) the situation between T2K and is reversed. If we have data from two long baseline experiments with different baseline then we can resolve the above degeneracy independent of the δ and sign of Δ_{31}.

The following method may be used to test the sign of Δ_{31}. From the experiment we will get three piece of data of three different neutrino oscillation experiment (T2K, $NO\nu A$, Double CHOOZ). Ne(T2K), Ne($NO\nu A$) and Ne(Double CHOOZ) is the data of three different experiments. We tested, whether the hypothesis of positive or negative Δ_{31} fit the data better. These number will be the function of θ_{13} and δ which as yet unknown. We compute $P^m_{\mu e}$ numerically by diagonalizing the matter dependent mass squared matrix foe each energy bin. In next section, we discuss the testing of Δ_{31} sign by χ^2 analysis.

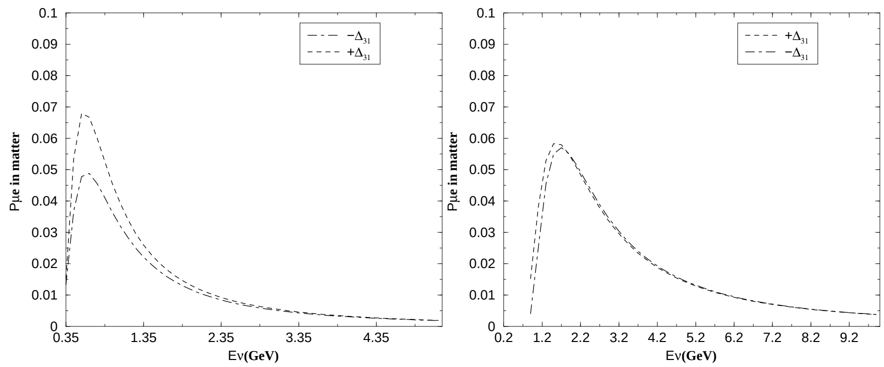

Figure 4: $P_{\mu e}$ oscillations probabilities vs E for $\Delta_{21} = 2.5 \times 10^{-3} eV^2$,$\theta_{13} = 10^o$,L=295 km and L=810 km. The dashed line is $P^m_{\mu e}(\delta = -90^o)$ with Δ_{31} positive and dot-dashed line is $P^m_{\mu e}(\delta = 90^o)$ with Δ_{31} negative.

5 Determine the Δ_{31} Sign by χ^2 Analysis

We take the combined data from Double CHOOZ [26], T2K [27] and $NO\nu A$ [28] can resolve the sign of Δ_{31}. This resolution depends crucially on matter effects which in turn depend on θ_{13}. If θ_{13} is unmeasurable small, it is extremely difficult to determine the sign of Δ_{31}. Here we address the question: what is the smallest value of θ_{13} for which the sign of Δ_{31} can be resolved by the data of the above three experiments, independent of the value of δ.

Since there is no data yet from any of these three experiments, we simulate data for each experiment. In our calculation we fix the values of the following neutrino parameters: $\Delta_{21} = 8.0 \times 10^{-5} eV^2$, $\theta_{12} = 34^o$ and $\theta_{23} = 45^o$. First we take $|\Delta_{31}| = 2.5 \times 10^{-3} eV^2$. The presently allowed range for θ_{13} is 0^o to 15^o and that for the CP phase δ is $-180^o$to 180^o. We pick the true value for θ_{13} from its allowed range and similarly for δ.. We call these values θ_{13}^{true} and δ^{true}. We take $\Delta_{31}$to be positive and compute the expected number of events in each bin of each experiment for θ_{13}^{true} and δ^{true}. We smear the computed event distributions in energy using the energy resolution functions estimated by the respective collaborations. The data obtained after the energy smearing, we call to be our simulated data, which consists of 92 data point N_p^{simu}, $p = 1$, 92. .Now we take Δ_{31} to be negative but keep Δ_{31} the same. We choose test values for θ_{13} and δ which we call θ_{13}^{test} and δ^{test}.. With these as inputs, we compute theoretical values for the number of events in each bin of each experiment. Thus we get 92 theoretical expectations, $N_p^{test}, p = 1$, 92..We compute χ^2 between the simulated data and the theoretical values

$$\chi^2(\delta^{test}, \theta_{13}^{test}) = \sum_{p=1}^{92} \frac{(N_p^{sim} - N_p^{th})^2}{\sigma_p^2} \tag{20}$$

In the above discussion, $p = 1,\ 28$ are data of Double CHOOZ, $p = 29$ to 46

are data of T2K and $p = 47$ to 92 are data of $NO\nu A$. σ_p is the error in N_p^{sim}.It is the square root of the sum of squares of statistical and systematic errors. In calculating the statistical error the background contribution to it is taken into account. Following the above procedure we compute a set of $\chi^2(\delta^{test}, \theta_{13}^{test})$ for all allowed values of θ_{13}^{test} and δ^{test}. Since N_p^{sim} and N_p^{test} are calculated using different signs of Δ_{31}, we expect χ^2 in Esq. (20) to be large. But, χ^2 is a function of θ_{13}^{test} and δ^{test}. Because of the parameter degeneracies discussed in sec(4) it is possible to have small χ^2 for $\theta_{13}^{test} \neq \theta_{13}^{true}$ and $\delta^{test} \neq \delta^{true}$even if Δ_{31} values have opposite signs in the calculation of N_p^{sim} and N_p^{test}. In particular, we require θ_{13}^{true} to be large enough such that Double CHOOZ will be able to measure its value. If the minimum of $\chi^2(\delta^{test}, \theta_{13}^{test})$ is greater than 4.0, then the two signs of Δ_{31} are distinguishable at 95% CL for the given values of θ_{13}^{true} and δ^{true}. If the minimum $\chi^2(\delta^{test}, \theta_{13}^{test})$ is less than 4.0, then the two signs of Δ_{31} can not be distinguished at 95% CL for the given values of θ_{13}^{true} and δ^{true}. We repeat the calculation for other values of θ_{13}^{true} *and* δ^{true}. We look for values of θ_{13}^{true} for which the minimum of $\chi^2(\delta^{test}, \theta_{13}^{test})$ is greater than 4.0 for all allowed values of δ^{true}. The minimum of θ_{13}^{true} for which the above condition is satisfied, is the smallest value of θ_{13} for which sign of Δ_{31} and hence the neutrino mass hierarchy, can be determined irrespective of the value of the CP phase δ.

We take the Double CHOOZ data [26] is divided into 28 bins. The measurements of the near detector give us the unoscillated neutrino event rate in each bin. The expected measurement in the far detector, for each bin, is given by

$$\frac{dN^{far}}{dE_\nu} = \frac{dN^{near}}{dE_\nu} \times \left(\frac{L^{near}}{L^{far}}\right)^2 P\{(\overline{\nu_e} \to \overline{\nu_e})\}, |\Delta_{31}|, \theta_{13}^{true}, L^{far})\}. \tag{21}$$

For Double CHOOZ, the expected error in energy measurement is much smaller than the bin size. Therefore the energy resolution can be taken to be a Dirac delta function. Thus the simulated number of events per bin is given by the above equation.

We see that T2K data [27] is divided into 18 bins. The expected electron neutrino event rate, in each bin, is given by

$$\frac{dN_e}{dE_\nu} = \frac{dN_\mu}{dE_\nu} P_{\mu e}^m(+\Delta_{31}, \theta_{13}^{true}, \delta^{true}, L_{T2K}). \tag{22}$$

T2K collaboration estimates the error in reconstructing the neutrino energy to be 100 MeV. We take the energy resolution function $R(E_\nu, E_{mea})$ to be a Gaussian with $\sigma = 100MeV$..We obtain the smeared event rate per bin by

$$\frac{dN_e}{dE_{mea}}|_{sim} = \sum \frac{dN_e}{dE_\nu} R(E_\nu, E_{mea}) dE_\nu \tag{23}$$

Finally we take the $NO\nu A$ [28] data is divided into 46 bins, for each of the off-axis locations $0mrd$,$7mrd$ and $14mrd$. We consider one off-axis location at a time. As in the case of T2K, the expected electron neutrino event rate, in each bin, is given by

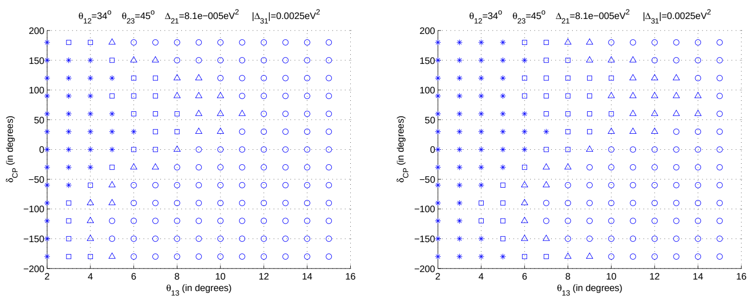

Figure 5: Plots of χ^2_{min} in $\theta_{13}^{true} - \delta^{true}$ plane, 0mr off-axis location with low energy (left) and medium energy (Right) options for $NO\nu A$ are assumed. $|\Delta_{31}| = 2.5 \times 10^{-3} eV^2$.The symbol are explained in the text.

$$\frac{dN_e}{dE_\nu} = \frac{dN_\mu}{dE_\nu} P^m_{\mu e}(+\Delta_{31}, \theta_{13}^{true}, \delta^{true}, L_{NO\nu A}). \tag{24}$$

Again as in the case of T2K, we obtain the smeared event distribution by means of a Gaussian resolution function with $\sigma = 100 MeV$. We assume that both T2K and $NO\nu A$ will run only in neutrino mode for five years. In computing the numbers for $NO\nu A$, we consider various different possibilities: Low energy beam with various different off-axis angles and also medium energy beam with various different off-axis angles. The theoretical expectation values are calculated using eq. (21), eq. (22) and eq. (24) with -Δ_{31},θ_{13}^{test} and δ^{test} as neutrino parameters. Note that no smearing is done in calculating theoretical expectation values for event numbers.

6 Summary

In this paper, we have studied the neutrino mass hierarchy. Our results are displayed in fig. (5), fig. (6) and fig. (7). In each figure we give a plot of χ^2_{min}in $\theta_{13}^{true} - \delta^{true}$ plane. The star symbol represents $\chi^2_{min} < 4.0$ square represents $4.0 < \chi^2_{min} < 9.0$ the triangle represents $9.0 < \chi^2_{min} < 16.0$ and circle represent $\chi^2_{min} > 16.0$. In each figure the left panel is generated assuming that $NO\nu A$will run in the low energy option and right panel is generated assuming high energy option. Fig.(5) corresponds to $0mrd$ off-axis location of $NO\nu A$, fig. (7) corresponds to $7mrd$ off-axis location and fig. (7) corresponds to $14mrd$ off-axis location. From the χ^2 analysis, we calculate the minimum value of θ_{13} for which the sign of Δ_{31} can be resolved at 95 %CL. For $|\Delta_{31}| = 2.5 \times 10^{-3} eV^2$ the low energy option with $0mrd$ and $7mrd$ off axis location seem to have the best resolving ability. We repeated our calculation for other allowed value of $|\Delta_{31}|$.In table 4, we compute the minimum value of θ_{13}^{true} for which the sign of Δ_{31} could be resolved at 95% CL, independent of the CP phase. We consider the $0mrd$, $7mrd$ and $14mrd$ off-axis angles of $NO\nu A$ for different values of Δ_{31}

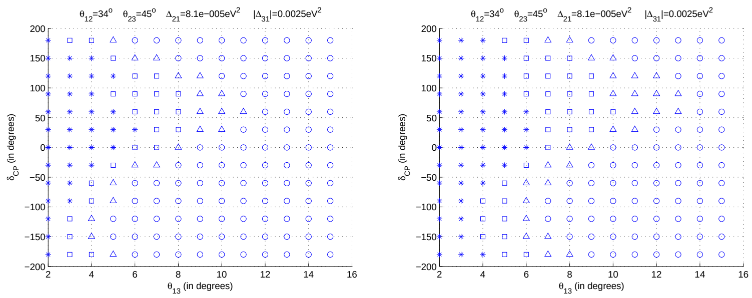

Figure 6: Plots of χ^2_{min} in $\theta_{13}^{true} - \delta^{true}$ plane, 7mr off-axis location with low energy (left) and medium energy (Right) options for $NO\nu A$ are assumed. $|\Delta_{31}| = 2.5 \times 10^{-3} eV^2$.The symbol are explained in the text.

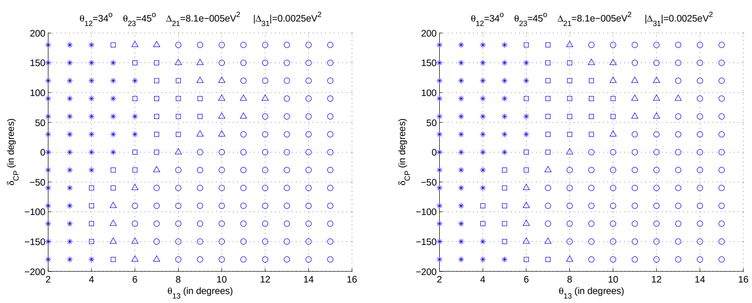

Figure 7: Plots of χ^2_{min} in $\theta_{13}^{true} - \delta^{true}$ plane, 14mr off-axis location with low energy (left) and medium energy (Right) options for $NO\nu A$ are assumed. $|\Delta_{31}| = 2.5 \times 10^{-3} eV^2$.The symbol are explained in the text.

$\lvert\Delta_{31}\rvert$	Minimum value of θ_{13}^{true}
$1.5 \times 10^{-3} eV^2$	15^o
$2.0 \times 10^{-3} eV^2$	9^o
$2.5 \times 10^{-3} eV^2$	7^o
$3.0 \times 10^{-3} eV^2$	4^o
$3.5 \times 10^{-3} eV^2$	4^o
$4.0 \times 10^{-3} eV^2$	4^o

Table 1: Minimum value of θ_{13}^{true}, for which the sign of Δ_{31} could be resolved at 95% CL, independent of CP phase.

From the table we see that the minimum value of θ_{13}^{true} for which the sign of Δ_{31}could be resolved at 95% CL independent of the CP phase. This minimum θ_{13}^{true} is the same for $0mrd$ and $7mrd$ off-axis angles of the low energy option of $NO\nu A$. The results are a little worse for the medium energy option of $NO\nu A$. Determining the type of neutrino mass hierarchy, whether normal or inverted, constitutes one of the fundamental question in neutrino physics. Future long baseline experiments aim at addressing this fundamental issue, but suffer typically from degeneracies with other neutrino parameters, namely $\theta_{13}$and δ The presence of such degeneracies limit the sensitivity to the type of hierarchy. Many earlier studies focused on the determination of the sign of $\Delta_{31}$by using the data of neutrinos and anti-neutrinos from more then one experiment [29, 30, 31, 32, 33]. In this present paper, we study the possibility of solving the neutrino mass hierarchy using only neutrino data of long baseline experiments T2K and $NO\nu A$ and data from Double CHOOZ. We determined, for each allowed value of $|\Delta_{31}|$, the minimum value of $\theta_{13}$for which the sign of Δ_{31} could be resolved independent of the value of the CP phase. If $|\Delta_{31}| = 0.0025eV^2$, we can rule out the wrong neutrino mass hierarchy at 95 % CL, for the whole range $\delta^{true} = -180^o - 180^o$, if $\theta_{13}^{true} \geq 7.0^o$. For larger values of $|\Delta_{31}|$ is less then $0.002eV^2$ the neutrino mass hierarchy can not be resolved by the data of the above three experiments for any of the allowed values of θ_{13}.

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